Delay-Induced Consensus and Quasi-Consensus in Multi-Agent Dynamical Systems

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***Abstract*—This paper studies consensus and quasi-consensus in multi-agent dynamical systems. A linear consensus protocol in the second-order dynamics is designed where both the current and delayed position information is utilized. Time delay, in a common perspective, can induce periodic oscillations or even chaos in dynamical systems. However, it is found in this paper that consensus and quasi-consensus in a multi-agent system cannot be reached without the delayed position information under the given protocol while they can be achieved with a relatively small time delay by appropriately choosing the coupling strengths. A necessary and sufficient condition for reaching consensus in multi-agent dynamical systems is established. It is shown that consensus and quasi-consensus can be achieved if and only if the time delay is bounded by some critical value which depends on the coupling strength and the largest eigenvalue of the Laplacian matrix of the network. The motivation for studying quasi-consensus is provided where the potential relationship between the second-order multi-agent system with delayed positive feedback and the first-order system with distributed-delay control input is discussed. Finally, simulation examples are given to illustrate the theoretical analysis.**

***Index Terms*—Algebraic graph theory, delay-induced consensus, multi-agent system, quasi-consensus.**

I. Introduction

COLLECTIVE behaviors in a group of autonomous mobile agents, e.g., synchronization [2], [21], [27], [36], [37], [41], [45], consensus [5]–[7], [15], [11], [12], [17], [19], [23], [28], [29], [32], [33], [38], [39], [42], [44], [46], formation control motion [3], [8], [25], swarming, and flocking [24], have been widely investigated recently due to the interest in animal group behaviors and broad applications in biological systems, sensor networks [43], UAV (Unmanned Air Vehicle) formations, robotic teams, underwater vehicles, etc. The main idea is that through a distributed protocol each agent shares information only with its neighbors while the whole network of agents simultaneously tries to coordinate with respect to certain global criteria of common interest. As a typical collective behavior, consensus usually refers to the problem of reaching an agreement among a group of autonomous agents, which serves as a basic foundation for the study of swarming and flocking behaviors.

Recently, many publications have been devoted to constructing conditions for reaching consensus among a group of autonomous agents in a dynamically changing environment. In [33], Vicsek *et al.* proposed a simple discrete-time model to study a group of autonomous agents moving in the plane with the same speed but different headings subject to noise perturbation, which in essence is the velocity consensus problem based on one of the heuristic rules proposed earlier by Reynolds [30]. Based on algebraic graph theory [9], the linear Vicsek's model was studied in [17] and it was found that consensus in a network with a switching topology can be reached if the network is jointly connected frequently enough as the network evolves with time. Afterwards, the study of consensus was further extended to the case of directed networks [5], [23].

In the literature, most existing works focused on the case where agents are governed by first-order dynamics [5], [6], [17], [23], [33]. However, second-order dynamics [11], [12], [28], [29], [32], [38], [40], [44] have also received increasing attention due to many real-world applications where agents are governed by both position and velocity dynamics. In [38], in particular, some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems with directed topologies were established. It was found that both the real and imaginary parts of the eigenvalues of the Laplacian matrix associated with the corresponding network topology play key roles in reaching consensus. However, as shown in [11], [12], [28], the velocity states of agents are often unavailable, therefore, some observers were designed with some additional variables involved, which leads to the study of higher-order dynamical systems.

It is well known that time delay, a destructive character in dynamics, may result in oscillatory behaviors [34], network instability (periodic oscillation and even chaos) [35], or the network desynchronization with a general coupling function [10], [16], [18]. On the other hand, consensus can be reached for any finite time delay on the neighboring agents in [20]. However, in [26], it was shown that time delay can induce system stability in linear time-invariant systems, where both the stability regions

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(pockets) in the domain of time delay and the number of unstable characteristic roots at any given pocket were theoretically analyzed. In [1], the delayed positive feedback was designed to stabilize the systems with second-order oscillations. Different from the results in [1], [26], quasi-consensus behavior is considered and the systems are coupled in this paper. In particular, the motivation for studying quasi-consensus is revealed where the potential relationship between the second-order multi-agent system with delayed positive feedback and the first-order system with distributed-delay control input is discussed. In [6], consensus in first-order multi-agent systems with current and outdated position states was discussed, showing that the delay-involved algorithm converges faster than the standard consensus protocol without time delays. In many real-world applications, the relative velocities of neighboring agents are difficult to be measured than relative positions [11], [12]. For example, a camera can be used for relative position measurements. In general, relative velocity measurements require more expensive sensors. In some experimental work, each mobile robot, equipped with range sensors, obtains the position information of its own and its neighbors through some localization algorithms. In the settings of such formation control problems with range-only sensing, the velocity information is difficult to be directly obtained. By using delayed position information in the memory and without knowing the velocity information of agents in second-order dynamics as in [11], [12], [29], it is first found in this paper that consensus can be reached by appropriately choosing network parameters while consensus may not be achieved without time delay. This implies that, similar to the delay-induced stability in linear time-invariant systems [26], time delay can induce consensus in multi-agent dynamical systems, which is the primary the motivation of the present work.

It should be emphasized that there does not exist physical communication delays in the network. This context essentially explores the combination of the current relative position with outdated relative position data (stored in memory) to help achieve consensus. As a result there is no need to measure relative velocities. In the designed system, the delays are not the REAL communication delays existing in the network but are outdated data stored in memory.

Note that a new consensus called quasi-consensus is defined in this paper where the velocity states of agents asymptotically converge to a common value but there are relative position differences among agents depending on the initial conditions, which is different from flocking [24] and formation control [3], [8], [25] in multi-agent systems. In [3], a behavior-based decentralized control for formation control architecture was proposed. Formation stabilization of a group of autonomous agents with linear dynamics was investigated by using structural potential functions in [25]. Then, a leader-follower problem for maintaining a desired formation was considered in [8]. For formation control and flocking in multi-agent systems, a geometrically desirable formation has been designed in prior while for quasi-consensus in this paper, the final position configuration changes with different initial states.

The main contribution of this paper is that a distributed protocol utilizing the current and delayed position information in multi-agent systems with second-order dynamics is designed which does not need the unavailable velocity information of agents. Then, a new concept for quasi-consensus in multi-agent systems under this setting is discussed. Some necessary and sufficient conditions are derived for reaching consensus, and it is found that consensus and quasi-consensus in multi-agent systems with both current and delayed position information can be reached if and only if the time delay is bounded by some critical values which depend on the coupling strengths and the largest eigenvalue of the Laplacian matrix of the network. Furthermore, the motivation for studying quasi-consensus is revealed where the potential relationship between the second-order multi-agent system with delayed positive feedback and the first-order system with distributed-delay control input is discussed.

The rest of the paper is organized as follows. In Section II, some preliminaries on graph theory and model formulation are given. The main results about delay-induced consensus and quasi-consensus in multi-agent dynamical systems are presented in Sections III. In Section IV, the motivation for introducing the quasi-consensus in multi-agent systems is discussed. Some numerical examples are given to illustrate the theoretical analysis in Section V. Conclusions are finally drawn in Section VI.

II. Preliminaries

In this section, some basic concepts and results about algebraic graph theory and model formulation are introduced.

A weighted undirected network $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ with order N consists of a set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, a set of undirected edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $G = (G_{ij})_{N\times N}$. An edge $\mathcal{E}_{ij}$ in a weighted undirected network $\mathcal{G}$ is denoted by the unordered pair of nodes (v_i, v_j), which means that nodes v_i and v_j can exchange information with each other. The weights $G_{ij} = G_{ji} > 0$ are positive if and only if there is an edge (v_i, v_j) in $\mathcal{G}$. A path between nodes v_i and v_j is a sequence of edges, $(v_i, v_{i_1}), (v_{i_1}, v_{i_2}), \ldots, (v_{i_l}, v_j)$, in the network with distinct nodes v_{i_k}, $k = 1, 2, \ldots, l$. An undirected network $\mathcal{G}$ is *connected* if there is a path between any pair of distinct nodes in $\mathcal{G}$.

For second-order dynamics, the consensus protocol in the literature is described by [28], [29], [38]

$$\begin{aligned} \dot{x}_i(t) &= v_i, \\ \dot{v}_i(t) &= \tilde{\alpha} \sum_{j=1, j\neq i}^{N} G_{ij}(x_j(t) - x_i(t)) \\ &\quad + \tilde{\beta} \sum_{j=1, j\neq i}^{N} G_{ij}(v_j(t) - v_i(t)), \\ &\qquad i = 1, 2, \ldots, N, \end{aligned} \tag{1}$$

where $x_i \in R^n$ and $v_i \in R^n$ are the position and velocity states of the ith agent (node), respectively, $\tilde{\alpha} > 0$ and $\tilde{\beta} > 0$ are the coupling strengths, $G = (G_{ij})_{N\times N}$ is the coupling configuration matrix representing the topological structure of the network and thus is the weighted adjacency matrix of the network, and the Laplacian matrix $L = (L_{ij})_{N\times N}$ is defined by

$$L_{ii} = -\sum_{j=1, j\neq i}^{N} L_{ij}, \quad L_{ij} = -G_{ij}, \quad i \neq j, \tag{2}$$

which ensures the diffusion property that $\sum_{j=1}^{N} L_{ij} = 0$. For notational simplicity, $n = 1$ is considered throughout the paper, but all the results obtained can be easily generated to the case with $n > 1$ by using the Kronecker product operations [14].

Definition 1: The multi-agent system is said to achieve *quasi-consensus* if for any initial conditions,

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = c_{ij}, \quad \lim_{t\to\infty} \|v_i(t) - v_j(t)\| = 0, \quad \forall i, j = 1, 2, \ldots, N,$$

where c_{ij} are constants. Particularly, if $c_{ij} = 0, \forall i, j = 1, 2, \ldots, N$, then the quasi-consensus is called *consensus*.

Since $\dot{x}_i(t) - \dot{x}_j(t) = v_i(t) - v_j(t)$ in (1), one only needs to check if the final velocity states of all the agents are the same for quasi-consensus. In [11], [12], [28], distributed observers were designed for dynamics of multi-agent systems where the velocity states were assumed to be unavailable, i.e., $\tilde{\beta} = 0$, and some slack variables were introduced and a higher-order controller was designed. In this paper, by using delayed position information, it will be shown that consensus and quasi-consensus can be reached in the multi-agent systems. To do so, the following consensus protocol with both current and delayed position information is considered

$$\begin{aligned} \dot{x}_i(t) &= v_i, \\ \dot{v}_i(t) &= \alpha \sum_{j=1, j\neq i}^{N} G_{ij}(x_j(t) - x_i(t)) - \beta \sum_{j=1, j\neq i}^{N} G_{ij} \\ &\quad \times (x_j(t-\tau) - x_i(t-\tau)), \; i = 1, 2, \ldots, N, \end{aligned} \tag{3}$$

where $\tau \geq 0$ is a time delay, and α and β are the coupling strengths. Because of (2), this system can be equivalently rewritten as follows:

$$\begin{aligned} \dot{x}_i(t) &= v_i, \\ \dot{v}_i(t) &= -\alpha \sum_{j=1}^{N} L_{ij} x_j(t) + \beta \sum_{j=1}^{N} L_{ij} x_j(t-\tau), \\ &\qquad i = 1, 2, \ldots, N. \end{aligned} \tag{4}$$

Lemma 1: [13] The Laplacian matrix L of an undirected network is symmetric and positive semi-definite. Moreover, L has a simple eigenvalue 0 and all the other eigenvalues are positive if and only if the undirected network is connected.

The following notations will be used throughout the paper for simplicity. Let $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ be the N eigenvalues of the Laplacian matrix L, $I_m \in R^{m\times m}$ $(O_N \in R^{m\times m})$ be a matrix with all entries being 1 (0), $1_m \in R^m$ $(0_N \in R^m)$ be a vector with all entries being 1 (0), $|a_1 + \mathbf{i}a_2| = \sqrt{a_1^2 + a_2^2}$ be the norm of a complex number $a_1 + \mathbf{i}a_2$ where $\mathbf{i} = \sqrt{-1}$, $\|x\| = (\sum_{j=1}^{m} |x_j|^2)^{1/2}$ be the norm of a complex vector $x = (x_1, \ldots, x_m)^T$, and $\mathcal{R}(u)$ and $\mathcal{I}(u)$ be the real and imaginary parts of a complex number u.

III. Delay-Induced Consensus and Quasi-Consensus in Multi-Agent Dynamical Systems

Let $\eta_i = (x_i, v_i)^T$, $C = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, and $D = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. Then, network (4) can be rewritten as

$$\dot{\eta}_i(t) = C\eta_i(t) - \alpha \sum_{j=1}^{N} L_{ij} D\eta_j(t) + \beta \sum_{j=1}^{N} L_{ij} D\eta_j(t-\tau), \quad i = 1, 2, \ldots, N. \tag{5}$$

Note that a solution of an isolated node satisfies

$$\dot{s}(t) = Cs(t), \tag{6}$$

where $s(t) = (s_1, s_2)^T$ is the state vector. Let $\eta = (\eta_1^T, \ldots, \eta_N^T)^T$ and rewrite system (5) into a matrix form:

$$\begin{aligned} \dot{\eta}(t) = [(I_N \otimes C) - \alpha(L \otimes D)]\eta(t) \\ + \beta(L \otimes D)\eta(t-\tau), \end{aligned} \tag{7}$$

where $\otimes$ is the Kronecker product [14]. Let Λ be the diagonal form associated with matrix L, i.e., there exists an unitary matrix P such that $P^T L P = \Lambda$, where $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$. Then, one has

$$\begin{aligned} &(P^T \otimes I_2)\dot{\eta}(t) \\ &= [(P^T \otimes I_2)(I_N \otimes C) - \alpha(\Lambda \otimes D)(P^T \otimes I_2)]\eta(t) \\ &\quad + \beta(\Lambda \otimes D)(P^T \otimes I_2)\eta(t-\tau) \\ &= [(P^T \otimes C) - \alpha(\Lambda \otimes D)(P^T \otimes I_2)]\eta(t) \\ &\quad + \beta(\Lambda \otimes D)(P^T \otimes I_2)\eta(t-\tau) \\ &= [(I_N \otimes C) - \alpha(\Lambda \otimes D)](P^T \otimes I_2)\eta(t) \\ &\quad + \beta(\Lambda \otimes D)(P^T \otimes I_2)\eta(t-\tau). \end{aligned}$$

Let $\xi(t) = (P^T \otimes I_2)\eta(t) = (\xi_1^T, \ldots, \xi_N^T)^T$, $\xi_i = (\xi_{i1}, \xi_{i2})^T$, $x = (x_1, \ldots, x_N)^T$, $v = (v_1, \ldots, v_N)^T$, $\tilde{\xi}_1 = (\xi_{11}, \ldots, \xi_{N1})^T$, and $\tilde{\xi}_2 = (\xi_{12}, \ldots, \xi_{N2})^T$. Then, the above multi-agent system can be transformed to

$$\begin{aligned} \dot{\xi}(t) = [(I_N \otimes C) - \alpha(\Lambda \otimes D)]\xi(t) \\ + \beta(\Lambda \otimes D)\xi(t-\tau), \end{aligned} \tag{8}$$

or

$$\dot{\xi}_i(t) = (C - \alpha\lambda_i D)\xi_i(t) + \beta\lambda_i D\xi_i(t-\tau), \quad i = 1, \ldots, N. \tag{9}$$

Theorem 1: Suppose that the network $\mathcal{G}$ is connected. Quasi-consensus in the multi-agent system (3) can be reached if and only if, in (8) or (9),

$$\lim_{t\to\infty} \|\xi_{i2}\| = 0, \; i = 2, \ldots, N. \tag{10}$$

Proof: (Sufficiency). Since the network is connected, $p_1 = 1_N/\sqrt{N}$ is the unit eigenvector of the Laplacian matrix L associated with the simple zero eigenvalue $\lambda_1 = 0$, where $LP =$

$P\Lambda$ and $P = (p_1, \ldots, p_N)$. Since $\lim_{t\to\infty} \|\xi_{i2}\| = 0$ for $i = 2, \ldots, N$ and $v(t) = P\tilde{\xi}_2(t) = \sum_{j=1}^N p_i\xi_{i2} \to p_1\xi_{i2}$, one has

$$\lim_{t\to\infty} \left\| v(t) - \frac{1}{\sqrt{N}}(\xi_{12}(t)^T, \ldots, \xi_{12}(t)^T)^T \right\| = 0,$$

where $\dot{\xi}_1(t) = C\xi_1(t)$.

(Necessity). If quasi-consensus in the multi-agent system (3) can be reached, then there exists a value $v^*(t) \in R$ such that $\lim_{t\to\infty} \|v(t) - 1_N \otimes v^*(t)\| = 0$. Since $0_N = P^T L 1_N = \Lambda P^T 1_N = (\lambda_1 p_1^T 1_N, \ldots, \lambda_N p_N^T 1_N)^T$, one has $p_i^T 1_N = 0$ for $i = 2, \ldots, N$. Therefore, $\|\xi_{i2}(t)\| = \|p_i^T v(t)\| \to \|(p_i^T 1_N) \otimes v^*(t)\| \to 0$, as $t \to \infty$ for all $i = 2, \ldots, N$.

Corollary 1: Suppose that the network $\mathcal{G}$ is connected. Quasi-consensus in the multi-agent system (3) can be reached if and only if each of the following $N-1$ equations

$$\lambda^2 + \lambda_i(\alpha - \beta e^{-\lambda\tau}) = 0,\ i = 2, \ldots, N, \tag{11}$$

has a simple zero root and the real parts of all the other roots are negative.

Proof: It suffices to prove that $\lim_{t\to\infty} \|\xi_{i2}\| = 0$, $i = 2, \ldots, N$, if and only if each of the equations in (11) has a simple zero root and the real parts of all the other roots are negative. The characteristic equation of the multi-agent system (9) is

$$\begin{aligned} &\det(\lambda I_2 - C + \alpha\lambda_i D - \beta\lambda_i e^{-\lambda\tau} D) \\ &= \det \begin{pmatrix} \lambda I_n & -I_n \\ (\alpha - \beta e^{-\lambda\tau})\lambda_i I_n & \lambda I_n \end{pmatrix} \\ &= \lambda^2 + (\alpha - \beta e^{-\lambda\tau})\lambda_i,\ i = 2, \ldots, N. \end{aligned} \tag{12}$$

(Sufficiency). If each of the equations in (11) has a simple zero root and the real parts of all the other roots are negative, then the states in (9) converge to some constants. Suppose that $\lim_{t\to\infty} \|\xi_{i2}(t)\| = \tilde{c}_i \neq 0$. Then it follows that $\lim_{t\to\infty} \|\xi_{i1}(t)\| = \infty$, which is a contradiction.

(Necessity). From $\lim_{t\to\infty} \|\xi_{i2}(t)\| = 0$ and $\dot{\xi}_{i1}(t) = \xi_{i2}(t)$, one knows that $\lim_{t\to\infty} \xi_{i1}(t) = c_i$, $i = 2, \ldots, N$, where c_i are constants. If each of the equations in (11) has at least one nonzero root with nonnegative real part, then $\xi_{i1}(t)$ or $\xi_{i2}(t)$ cannot converge; or if one of the equations in (11) has more than one zero root, then one has $\alpha = \beta = 0$ or $\alpha = \beta$ and $\tau = 0$. In both cases, $\xi_{i1}(t)$ or $\xi_{i2}(t)$ cannot converge. □

Corollary 2: Suppose that the network $\mathcal{G}$ is connected. Consensus in the multi-agent system (3) can be reached if and only if, in (9) or (9),

$$\lim_{t\to\infty} \|\xi_i\| = 0,\ i = 2, \ldots, N,$$

or equivalently if and only if the real parts of all the roots in (11) are negative.

Proof: The result can be proved through examining the state $x(t)$ following the same process as in the proofs of Theorem 1 and Corollary 1.

Some necessary and sufficient conditions for reaching consensus or quasi-consensus in the multi-agent system (3) have been obtained in Corollaries 1 and 2 above. Next, we will show that consensus and quasi-consensus in multi-agent system (3) cannot be achieved when $\tau = 0$; however, they can be reached by appropriately choosing the time delay τ and the coupling strengths α and β.

Lemma 2: Suppose that the network $\mathcal{G}$ is connected. Consensus and quasi-consensus in the multi-agent systems (3) cannot be reached when $\tau = 0$. However, for a sufficiently small $\tau > 0$ and given fixed control gains α and β, consensus (resp. quasi-consensus) can be reached if and only if $\alpha > \beta > 0$ (resp. $\alpha = \beta > 0$).

Proof: From (11), one has $\lambda = \pm\sqrt{\lambda_i(\beta - \alpha)}$ when $\tau = 0$. If $\alpha = \beta$, each of the (11) has two zero roots; if $\alpha \neq \beta$, there exits at least one nonzero root with nonnegative real part. Therefore, consensus and quasi-consensus in the multi-agent systems (3) cannot be reached if $\tau = 0$.

From (11), one has $\lambda^2 = \lambda_i(\beta e^{-\lambda\tau} - \alpha)$, $i = 2, \ldots, N$. If $\mathcal{R}(\lambda) \geq 0$, then $|e^{-\lambda\tau}| \leq 1$. Thus, it follows that $|\lambda|^2 \leq \lambda_i(|\beta| + \alpha)$, which indicates that $|\lambda|$ is bounded. If $\mathcal{R}(\lambda) < 0$, the orders of λ^2 and $e^{-\lambda\tau}$ with regard to λ are different when $\tau > 0$, and thus $|\lambda|$ is bounded. For a sufficiently small τ, one obtains

$$\begin{aligned} \lambda^2 &= \lambda_i \left[\beta(1 - \lambda\tau) + \beta\frac{(\lambda\tau)^2}{2} - \alpha + o(\tau^2) \right] \\ &= \lambda_i \left[\frac{\beta\tau^2}{2}\lambda^2 - \beta\tau\lambda + (\beta - \alpha) + o(\tau^2) \right]. \end{aligned}$$

It follows that

$$\left(1 - \frac{\lambda_i\beta\tau^2}{2}\right)\lambda^2 + \lambda_i\beta\tau\lambda + \lambda_i(\alpha - \beta) + o(\tau^2) = 0. \tag{13}$$

By Lemma 1, one has $\lambda_i > 0$, $i = 2, \ldots, N$. From (11), it is easy to see that zero is a simple root if and only if $\alpha = \beta \neq 0$ since $\lambda^2 + \lambda_i\alpha(1 - e^{-\lambda\tau}) = 0$ when $\lambda = 0$. If $\alpha = \beta > 0$, then $\lambda \approx 0$ or $\lambda \approx -(\lambda_i\beta\tau)/(1 - (\lambda_i\beta\tau^2)/(2)) < 0$. Therefore, quasi-consensus can be reached for a sufficiently small $\tau > 0$ if and only if $\alpha = \beta > 0$.

From (13), one obtains

$$\lambda \approx \frac{-\lambda_i\beta\tau \pm \sqrt{(\lambda_i\beta\tau)^2 - 4\left(1 - \frac{\lambda_i\beta\tau^2}{2}\right)\lambda_i(\alpha - \beta)}}{2\left(1 - \frac{\lambda_i\beta\tau^2}{2}\right)}.$$

The real parts of all the roots in (11) are negative if and only if $\alpha > \beta > 0$ where $(\lambda_i\beta\tau)^2 - 4(1 - (\lambda_i\beta\tau^2)/(2))\lambda_i(\alpha - \beta) < 0$ for a sufficiently small $\tau > 0$.

Remark 1: It is easy to see from Lemma 2 that consensus (resp. quasi-consensus) in the multi-agent systems (3) cannot be reached without delay, i.e., $\tau = 0$, but interestingly they can be reached even for a sufficiently small $\tau > 0$ by choosing some appropriate coupling strengths $\alpha > \beta > 0$ (resp. $\alpha = \beta > 0$). It is well known that the time delay may result in oscillatory behaviors or network instability (periodic oscillation and chaos) [35]. However, as shown by Lemma 2 above, time delay here can induce consensus in the multi-agent system (3). Moreover, in order to reach consensus in the multi-agent system (3), the coupling strength of the current states should be larger than that of the outdated states, i.e., $\alpha > \beta$, at the nodes of the network.

Lemma 3: Suppose that the network $\mathcal{G}$ is connected. Each of the equations in (11) has a purely imaginary root if and only if

$$\tau = \frac{k\pi}{\sqrt{\lambda_i(\alpha \pm \beta)}}, \quad \text{when } \alpha > \beta > 0; \quad k = 1, 2, \ldots; \quad i = 2, \ldots, N, \tag{14}$$

or if and only if

$$\tau = \frac{k\pi}{\sqrt{\lambda_i(\alpha + \beta)}}, \text{ when } \alpha = \beta > 0; \quad k = 1, 2, \ldots; \ i = 2, \ldots, N. \tag{15}$$

Proof: Let $\lambda = \mathbf{i}\omega$. Without loss of generality, suppose that $\omega > 0$. From (11), one has

$$\omega^2 = \lambda_i(\alpha - \beta e^{-\mathbf{i}\omega\tau}). \tag{16}$$

Separating the real and imaginary parts of (16) yields

$$\omega\tau = k\pi, \ k = 1, 2, \ldots$$

It follows that $\cos(\omega\tau) = \pm 1$. If $\alpha > \beta > 0$, then $\omega^2 = \lambda_i(\alpha \pm \beta)$. Since $\omega > 0$, one has $\omega^2 = \lambda_i(\alpha + \beta)$ when $\alpha = \beta > 0$. □

Lemma 4: [22], [31] Consider the exponential polynomial

$$\begin{aligned} P(\lambda, e^{-\lambda\tau_1}, \ldots, e^{-\lambda\tau_m}) &= \lambda^n + p_1^{(0)}\lambda^{n-1} + \cdots + p_{n-1}^{(0)}\lambda + p_n^{(0)} \\ &+ \left[p_1^{(1)}\lambda^{n-1} + \cdots + p_{n-1}^{(1)}\lambda + p_n^{(1)}\right] e^{-\lambda\tau_1} + \cdots \\ &+ \left[p_1^{(m)}\lambda^{n-1} + \cdots + p_{n-1}^{(m)}\lambda + p_n^{(m)}\right] e^{-\lambda\tau_m}, \end{aligned}$$

where $\tau_i \geq 0$ $(i = 1, 2, \ldots, m)$ and $p_j^{(i)}(i = 0, 1, \ldots, m; j = 1, 2, \ldots, n)$ are constants. As $(\tau_1, \tau_2, \ldots, \tau_m)$ are varied, the sum of the orders of the zeros of $P(\lambda, e^{-\lambda\tau_1}, \ldots, e^{-\lambda\tau_m})$ on the open right-half plane can change only if a zero appears on or across the imaginary axis.

Lemma 5: Suppose that the network $\mathcal{G}$ is connected. Let λ be a solution in (11). Then,

$$\mathcal{R}\left(\frac{d\lambda}{d\tau}\right)\bigg|_{\tau = \frac{k\pi}{\sqrt{\lambda_i(\alpha+\beta)}}} > 0, \quad \alpha \geq \beta > 0; \quad k = 1, 2, \ldots; i = 2, \ldots, N. \tag{17}$$

Proof: Let $f_i(\lambda, \tau) = \lambda^2 + \alpha\lambda_i - \beta\lambda_i e^{-\lambda\tau}$, $\tau_{ki} = (k\pi)/(\sqrt{\lambda_i(\alpha + \beta)})$, and $\omega_i = \sqrt{\lambda_i(\alpha + \beta)}$. Then, from Lemma 4, one has $f_i(\mathbf{i}\omega_i, \tau_{ki}) = 0$. Since $f_i(\lambda, \tau)$ is continuous around the point $(\mathbf{i}\omega_i, \tau_{ki})$, $(\partial f_i)/(\partial\lambda)$ and $(\partial f_i)/(\partial\tau)$ are continuous, and $(\partial f_i)/(\partial\lambda)|_{(\mathbf{i}\omega_i, \tau_{ki})} \neq 0$, λ is differentiable with respect to τ around the point $(\mathbf{i}\omega_i, \tau_{ki})$ according to the implicit function theorem.

Taking the derivative of λ with respect to τ in $f_i(\lambda, \tau) = 0$, one obtains

$$2\lambda\frac{d\lambda}{d\tau} + \beta\lambda_i\left(\lambda + \tau\frac{d\lambda}{d\tau}\right)e^{-\lambda\tau} = 0. \tag{18}$$

It follows that

$$\begin{aligned} \frac{d\lambda}{d\tau} &= -\frac{\beta\lambda_i\lambda e^{-\lambda\tau}}{2\lambda + \beta\lambda_i\tau e^{-\lambda\tau}} \\ &= -\beta\lambda_i\frac{\lambda}{2\lambda e^{\lambda\tau} + \beta\lambda_i\tau} \\ &= -\frac{\beta\lambda_i(\mathcal{R}(\lambda) + \mathbf{i}\mathcal{I}(\lambda))}{\theta(\lambda, \tau)}. \end{aligned}$$

where $\theta(\lambda, \tau) = 2e^{\mathcal{R}(\lambda)\tau}\{[\mathcal{R}(\lambda)\cos(\mathcal{I}(\lambda)\tau) - \mathcal{I}(\lambda)\sin(\mathcal{I}(\lambda)\tau)] + \mathbf{i}[\mathcal{R}(\lambda)\sin(\mathcal{I}(\lambda)\tau) + \mathcal{I}(\lambda)\cos(\mathcal{I}(\lambda)\tau)]\} + \beta\lambda_i\tau$.

Let

$$\begin{aligned} q(\lambda, \tau) &= \Big(2e^{\mathcal{R}(\lambda)\tau}[\mathcal{R}(\lambda)\cos(\mathcal{I}(\lambda)\tau) \\ &\quad - \mathcal{I}(\lambda)\sin(\mathcal{I}(\lambda)\tau)] + \beta\lambda_i\tau\Big)^2 \\ &\quad + \Big(2e^{\mathcal{R}(\lambda)\tau}[\mathcal{R}(\lambda)\sin(\mathcal{I}(\lambda)\tau) \\ &\quad + \mathcal{I}(\lambda)\cos(\mathcal{I}(\lambda)\tau)]\Big)^2 \geq 0. \end{aligned}$$

By simple calculations, one has

$$\begin{aligned} &-\frac{q(\lambda, \tau)}{\beta\lambda_i}\mathcal{R}\left(\frac{d\lambda}{d\tau}\right) \\ &= \mathcal{R}(\lambda)\Big(2e^{\mathcal{R}(\lambda)\tau}[\mathcal{R}(\lambda)\cos(\mathcal{I}(\lambda)\tau) \\ &\quad - \mathcal{I}(\lambda)\sin(\mathcal{I}(\lambda)\tau)] + \beta\lambda_i\tau\Big) \\ &\quad + \mathcal{I}(\lambda)\Big(2e^{\mathcal{R}(\lambda)\tau}[\mathcal{R}(\lambda)\sin(\mathcal{I}(\lambda)\tau) \\ &\quad + \mathcal{I}(\lambda)\cos(\mathcal{I}(\lambda)\tau)]\Big) \\ &= \mathcal{R}(\lambda)\beta\lambda_i\tau + 2e^{\mathcal{R}(\lambda)\tau}|\lambda|^2\cos(\mathcal{I}(\lambda)\tau). \end{aligned}$$

If $\tau = \tau_{ik}$, then $\lambda = \mathbf{i}\omega_i$ and $\cos(\mathcal{I}(\lambda)\tau) = -1$. Finally, one gets

$$\mathcal{R}\left(\frac{d\lambda}{d\tau}\right)\bigg|_{\tau = \frac{k\pi}{\sqrt{\lambda_i(\alpha+\beta)}}} = \frac{2\beta\lambda_i^2(\alpha + \beta)}{\beta^2\lambda_i^2\tau^2 + 4(\alpha + \beta)} > 0.$$

□

Theorem 2: Suppose that the network $\mathcal{G}$ is connected.

1) Consensus can be reached in the multi-agent system (3) if and only if

$$\frac{2k\pi}{\sqrt{\lambda_i(\alpha - \beta)}} < \tau < \frac{(2k+1)\pi}{\sqrt{\lambda_i(\alpha + \beta)}}, \ \alpha > \beta > 0, \tag{19}$$

for all $i = 2, \ldots, N$ where $k = 0, 1, \ldots$.

2) Quasi-consensus can be reached in the multi-agent system (3) if and only if

$$0 < \tau < \frac{\pi}{\sqrt{\lambda_N(\alpha + \beta)}}, \ \alpha = \beta > 0. \tag{20}$$

Proof:

1) The proof can be analyzed from Nyquist criterion by using frequency domain approach. Equation (11) can be written in a frequency domain form [1]:

$$1 - G(s) = 1 - \frac{\lambda_i\beta e^{-s\tau}}{s^2 + \lambda_i\alpha} = 0, i = 2, \ldots, N. \tag{21}$$

Note that if $\tau = 0$, the Nyquist plot always encircles the point $(-1, 0)$. Therefore, a necessary condition for the stability of (22) is that $\alpha > \beta > 0$. Otherwise, there is at least one clockwise encirclement in $(-1, 0)$ with Nyquist plot. One only needs to consider the number of encirclements of $(-1, 0)$ by Nyquist plot:

$$-G(\mathbf{i}\omega) = \frac{-\lambda_i \beta e^{-\mathbf{i}\omega\tau}}{\lambda_i \alpha - \omega^2}\, i = 2, \ldots, N. \tag{22}$$

Consider all the intersections of the polar plot with the negative real axis. Then, one has that there are no encirclements at $(-1, 0)$ of the Nyquist plot if and only if [1]:

$$\frac{2k\pi}{\sqrt{\lambda_i(\alpha - \beta)}} < \tau < \frac{(2k+1)\pi}{\sqrt{\lambda_i(\alpha + \beta)}}, i = 2, \ldots, N, \tag{23}$$

where $k = 0, 1, \ldots$.

2) Since the network $\mathcal{G}$ is connected and from Lemma 2, one knows that quasi-consensus can be reached for a sufficiently small $\tau > 0$ if and only if $\alpha = \beta > 0$. Consider τ as a parameter varying from 0 to $+\infty$. By Lemma 3, a purely imaginary root first emerges when $\tau = (\pi)/(\sqrt{\lambda_N(\alpha + \beta)})$. In view of Lemmas 4 and 5, (11) has a simple zero root and the real parts of all the other roots are negative if $0 < \tau < (\pi)/(\sqrt{\lambda_N(\alpha + \beta)})$ and $\alpha = \beta > 0$, and there is at least one nonzero root with nonnegative real part if $\tau \geq (\pi)/(\sqrt{\lambda_N(\alpha + \beta)})$. Therefore, quasi-consensus can be reached in the multi-agent system (3) if and only if $0 < \tau < (\pi)/(\sqrt{\lambda_N(\alpha + \beta)})$ and $\alpha = \beta > 0$. □

For a fixed network topology, consensus can be reached in the multi-agent system (3) if and only if τ is bounded by some critical values by choosing appropriate coupling strengths $\alpha > \beta > 0$. On the other hand, when the time delay τ is fixed, an interesting problem is how to design the coupling strengths such that consensus can be reached. This issue is addressed by the following result.

Corollary 3: Suppose that the network $\mathcal{G}$ is connected. Consensus (quasi-consensus) can be reached in the multi-agent system (3) if

$$\alpha + \beta < \frac{\pi^2}{\lambda_N \tau^2}, \, \alpha > \beta > 0 \, (\alpha = \beta > 0). \tag{24}$$

Remark 2: In the multi-agent system (3), the velocity states of the agents are updated based on the current and delayed position states of their neighboring agents. If the control gain $\alpha + \beta$ is very large, then from the conditions in Theorem 2 and Corollary 3, the allowable time delay should be very small such that the delayed information can follow the states of neighboring agents in real time. However, if the time delay is large, this delayed position information may be outdated therefore cannot reflect the real time states of neighboring agents. Thus, the larger the coupling strength $\alpha + \beta$ is, the smaller the time delay τ should be.

IV. Motivation for Quasi-Consensus

In the above section, quasi-consensus in multi-agent system (3) is introduced. In order to motivate the idea for defining the new concept quasi-consensus in multi-agent systems with second-order dynamics, it is very interesting to see that the studied model is the exact first-order multi-agent system with the control input involving the distributed delay. Actually, in the multi-agent system (3), each agent needs some memory to store the outdated information of its neighboring agents.

Next, a typical multi-agent system with memory of distributed delay is considered:

$$\dot{y}_i(t) = -\gamma \int_{t-\tau}^{t} \sum_{j=1}^{N} L_{ij} y_j(z) dz, \, i = 1, 2, \ldots, N, \tag{25}$$

where $y_i \in R^n$ is the state of agent i, L_{ij} is defined as above, and $\gamma > 0$ is the coupling strength. If the initial condition for (25) is well defined such that $\dot{y}_i(t)$ is differentiable, then one has

$$\begin{aligned} \dot{y}_i(t) &= u_i(t) \\ \dot{u}_i(t) &= -\gamma \sum_{j=1}^{N} L_{ij} y_j(t) + \gamma \sum_{j=1}^{N} L_{ij} y_j(t - \tau), \\ & \qquad i = 1, 2, \ldots, N, \end{aligned} \tag{26}$$

which is exactly system (3) or (4) with $\alpha = \beta = \gamma$.

Corollary 4: Suppose that the network $\mathcal{G}$ is connected. Quasi-consensus can be reached in the multi-agent system (25) if and only if

$$0 < \tau < \frac{\pi}{\sqrt{2\gamma\lambda_N}}. \tag{27}$$

Proof: Choose $y_i = x_i$ and $\gamma = \alpha = \beta$ in (3). Then, the result in (26) can be easily obtained by Theorem 2. □

Remark 3: In the multi-agent system (25), only quasi-consensus can be reached if the time delay τ is less than a critical value $(\pi)/(\sqrt{2\gamma\lambda_N})$, and it should be noted here that consensus in (25) cannot be reached for any time delay $\tau > 0$ and any coupling strength γ. To satisfy the condition $\alpha > \beta$ for reaching consensus as in Theorem 2, a modified system of (25) is considered:

$$\dot{y}_i(t) = -\gamma \int_{t-\tau}^{t} \sum_{j=1}^{N} L_{ij} g(z, t) y_j(z) dz, \quad i = 1, 2, \ldots, N, \tag{28}$$

where $g(\cdot)$ is a weighting function. For example, one can choose $g(s, t) = (1)/(1 - e^{-\tau}) e^{-(t-z)}$ satisfying $\int_{t-\tau}^{t} g(z, t) = 1$ [36].

V. Simulation Examples

In this section, some simulation examples are given to verify the theoretical analysis.

A. Consensus and Quasi-Consensus in a Scale-Free Complex Network

A scale-free network is generated in the simulation, where the number of initial nodes is 5, and at each time step a new node is introduced and connected to 5 existing nodes in the network with degree preferential attachment, until the total number of nodes $N = 100$ [4]. By computation, one obtains $\lambda_N = 32.5389$. Let $\alpha = 1$ and $\beta = 0.5$. From Theorem 2, one knows that consensus can be reached in the multi-agent system

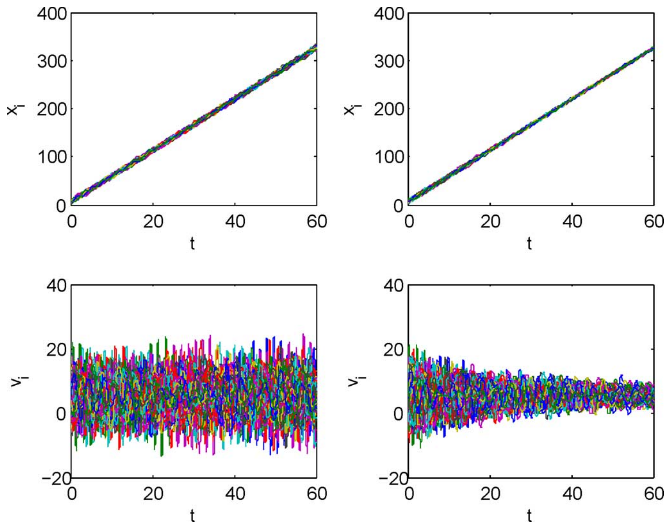

Fig. 1. Position and velocity states of agents in a multi-agent dynamical system with a scale-free network topology, where $\tau = 0$ (a: left) and $\tau = 0.01$ (b: right).

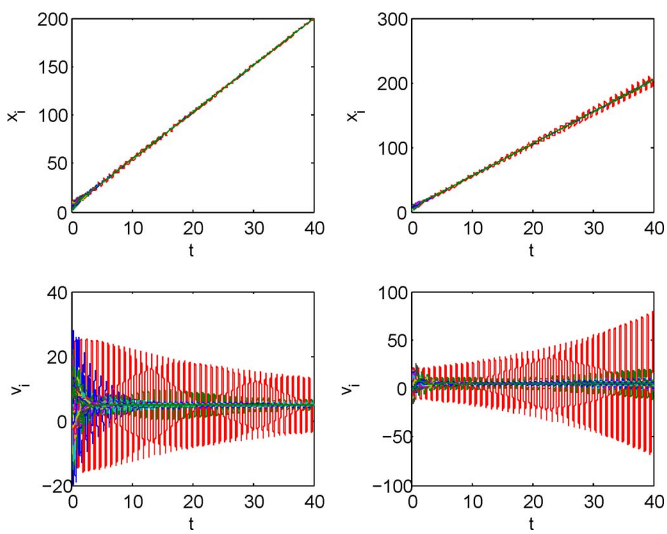

Fig. 2. Position and velocity states of agents in a multi-agent dynamical system with a scale-free network topology, where $\tau = 0.444$ [(a): left] and $\tau = 0.454$ [(b): right].

(3) if and only if $\tau \in (0, (\pi)/(\sqrt{\lambda_N(\alpha+\beta)})) = (0, 0.4497)$. The position and velocity states of all the agents are shown in Figs. 1 and 2, where consensus cannot be achieved when $\tau = 0$ [Fig. 1(a)] and $\tau = 0.454$ [Fig. 2(b)] but it can be reached if $\tau = 0.01$ [Fig. 1(b)] and $\tau = 0.444$ [Fig. 2(a)]. It is easy to see that the numerical simulations well confirm the theoretical analysis.

Actually, the real parts of all the roots in (11) are negative for $\tau \in (0, (\pi)/(\sqrt{\lambda_N(\alpha+\beta)})) = (0, 0.4497)$ in this example. It is quite easy to see that the convergence rate can be facilitated by choosing an appropriate time delay τ. By simple calculation, the time delay τ_m for reaching a fast convergence rate satisfies $\tau_m = \max_\tau \min_i(\mathcal{R}(\lambda))$, where $\lambda^2 + \lambda_i(\alpha - \beta e^{-\lambda\tau}) = 0$.

Consider the multi-agent system with the same network structure as above. Let $\alpha = 1$ and $\beta = 1$. From Theorem 2, one knows that quasi-consensus can be reached in the multi-agent system (3) if and only if $\tau \in (0, (\pi)/(\sqrt{\lambda_N(\alpha+\beta)})) = (0, 0.3894)$. The position and

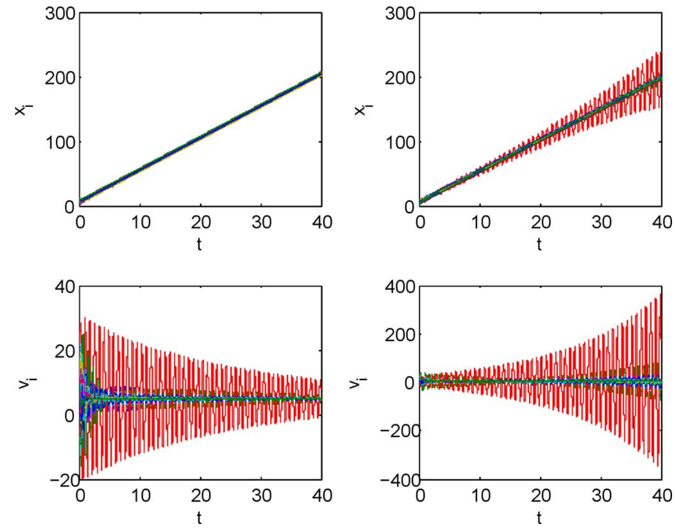

Fig. 3. Position and velocity states of agents in a multi-agent dynamical system with a scale-free network topology, where $\tau = 0.384$ [(a): left] and $\tau = 0.394$ [(b): right].

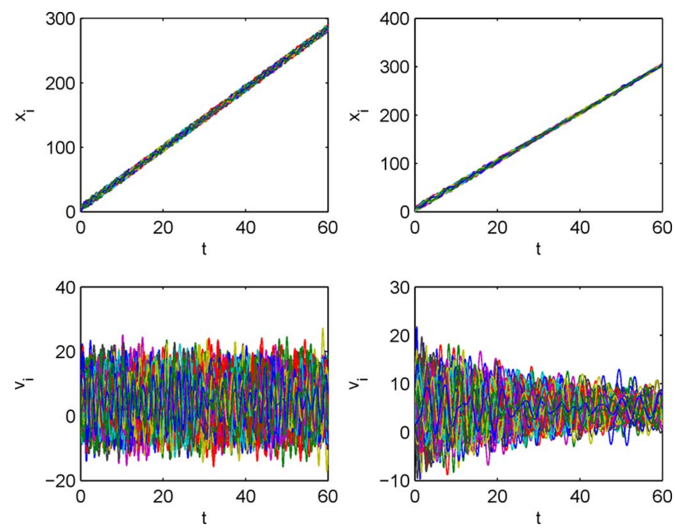

Fig. 4. Position and velocity states of agents in a multi-agent dynamical system with a random network topology, where $\tau = 0$ [(a): left] and $\tau = 0.01$ [(b): right].

velocity states of all the agents are shown in Fig. 3, where consensus cannot be achieved when $\tau = 0.394$ [Fig. 3(b)] but it can be reached if $\tau = 0.384$ [Fig. 3(a)].

B. Consensus and Quasi-Consensus in a Random Network

A random network is also performed in the simulation, where each pair of nodes is connected with the probability $p = 0.1$ and the total number of nodes $N = 100$. By simple calculation, one obtains $\lambda_N = 26.1441$. Let $\alpha = 1$ and $\beta = 0.5$. From Theorem 2, one knows that consensus can be reached in multi-agent system (3) if and only if $\tau \in (0, (\pi)/(\sqrt{\lambda_N(\alpha+\beta)})) = (0, 0.5017)$. The position and velocity states of all the agents are shown in Fig. 4. From Lemma 2, one knows that consensus cannot be achieved when $\tau = 0$ [Fig. 4(a)] while it can be reached if $\tau = 0.01$ [Fig. 4(b)], which indicates that a small time delay can induce consensus in multi-agent system (3).

In order to verify the quasi-consensus in multi-agent system (3), the same parameters $\alpha = 1$ and $\beta = 1$ are

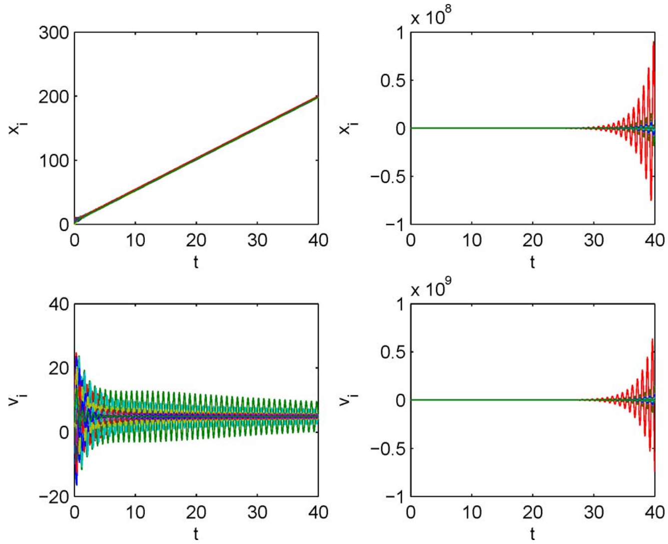

Fig. 5. Position and velocity states of agents in a multi-agent dynamical system with a random network topology, where $\tau = 0.43$ [(a): left] and $\tau = 0.44$ [(b): right].

considered. From Theorem 2, one knows that quasi-consensus can be reached in multi-agent system (3) if and only if $\tau \in (0, (\pi)/(\sqrt{\lambda_N(\alpha+\beta)})) = (0, 0.4345)$. The position and velocity states of all the agents are shown in Fig. 5, where consensus cannot be achieved when $\tau = 0.43$ [Fig. 5(b)] but it can be reached if $\tau = 0.44$ [Fig. 5(a)].

VI. Conclusion

In this paper, a linear consensus protocol with second-order dynamics has been designed based on both current and delayed position information of agents. The time delay, usually a destructive character in dynamics, can induce periodic oscillations and even chaos in dynamical systems. However, it has been found in this paper that consensus and quasi-consensus in a multi-agent system cannot be reached without time delay under the given protocol while they can be achieved with a relatively small time delay by appropriately choosing the network coupling strengths. A necessary and sufficient condition for reaching consensus has been derived, which shows that consensus and quasi-consensus can be achieved in a multi-agent system if and only if the time delay is bounded by some critical values depending on the coupling strengths and the largest eigenvalue of the Laplacian matrix in the network. The designed consensus protocol with both current and delayed position information is very useful especially when the velocity information of the neighboring agents is unavailable. The allowable maximum communication delay for reaching consensus has been theoretically analyzed, which is helpful for the design and implementation of collective behaviors in multi-agent systems.

There are still many related interesting problems deserving further investigations. For example, it is of interest to study the multi-agent systems with nonuniform time delays and general directed topologies, the critical time delays for reaching the fastest convergence, and more general protocols with negative weights in (28), which will be investigated in the future.

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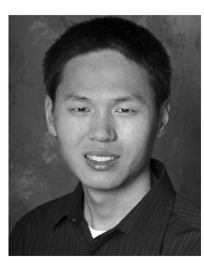

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